Multilevel Analysis: An Applied Introduction

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What are Multilevel Data?

- Data that are hierarchically structured, nested, clustered
- Data collected from units organized or observed within units at a higher level (from which data are also obtained)

data collected on	*who are clustered within*
students	classrooms
siblings	families
repeated observations	individuals

==> these are examples of two-level data

level 1 - (students) - measurement of primary outcome and important mediating variables

level 2 - (classrooms) - provides context or organization of level-1 units which may influence outcome; other mediating variables

What is Multilevel Data Analysis?

"any set of analytical procedures that involve data gathered from individuals and from the social structure in which they are embedded and are analyzed in a manner that models the multilevel structure"

L. Burstein, *Units of Analysis*, 1985, Int Ency of Educ

- analysis that *models the multilevel structure*
- recognizes influence of structure on individual outcome

structure	*may influence response from*
classroom	students
family	siblings
individual	repeated observations

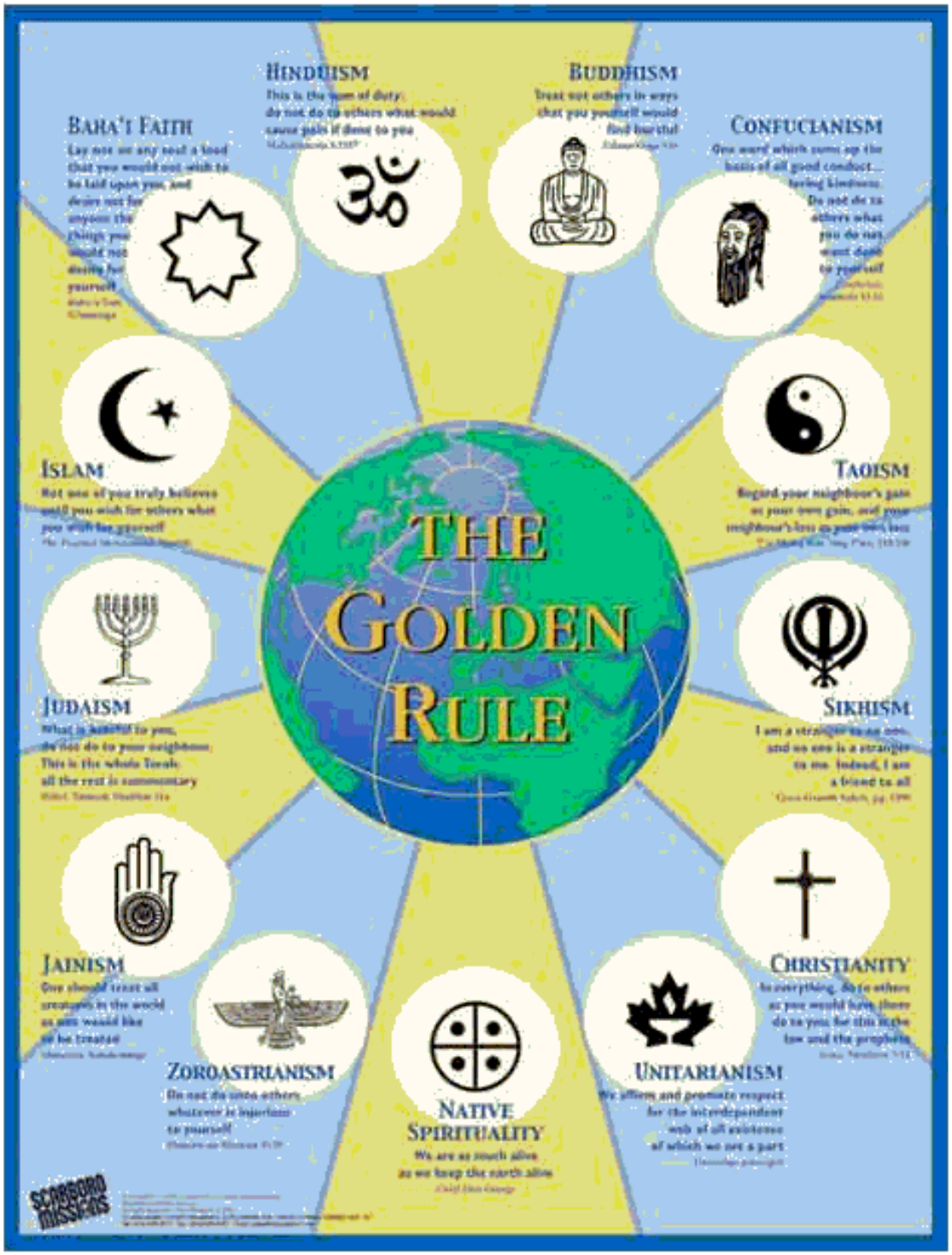
Why do Multilevel Data Analysis?

- assess amount of variability due to each level (*e.g.*, family variance and individual variance)
- model level 1 outcome in terms of effects at both levels

$$individual\ var. = fn(individual\ var. + family\ var.)$$

- assess interaction between level effects (*e.g.*, individual outcome influenced by family SES for males, not females)
- Responses are not independent - individuals within clusters share influencing factors

$\Rightarrow$ Multilevel analysis - another example of *Golden Rule of Statistics*: "one person's error term is another person's (or many persons') career"


THE GOLDEN RULE
HINDUISM
BUDDHISM
BARA'I FAITH
CONFUCIANISM
ISLAM
TAOISM
JUDAISM
SIKHISM
JAINISM
CHRISTIANITY
ZOROASTRIANISM
NATIVE SPIRITUALITY
We are as much alive as we keep the earth alive
UNITARIANISM
SCARBORO MISSIONS

		cluster variables		*subject variables*		
cluster	subject	tx group	size	outcome	sex	age
1	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_1	.	.	.	.	.
2	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_2	.	.	.	.	.
.	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	$n_.$	.	.	.	.	.
N	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_N	.	.	.	.	.

$i = 1 \ldots N$ clusters

$j = 1 \ldots n_i$ subjects in cluster i

		time-invariant variables			*time-varying variables*	
subject	time	tx group	sex	age	outcome	dose
1	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_1	.	.	.	.	.
2	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_2	.	.	.	.	.
.	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	$n_.$	.	.	.	.	.
N	1	.	.	.	.	.
	$\vdots$	.	.	.	.	.
	n_N	.	.	.	.	.

$i = 1 \ldots N$ subjects

$j = 1 \ldots n_i$ timepoints for subject i

Multilevel models aka

- random-effects models
- random-coefficient models
- mixed-effects models
- hierarchical linear models

Useful for analyzing

- Clustered data
  - subjects (level-1) within clusters (level-2)
    - e.g., clinics, hospitals, families, worksites, schools, classrooms, city wards
- Longitudinal data
  - repeated obs. (level-1) within subjects (level-2)

General (2-level) Model for Clustered Data

Consider the model with p covariates for the $n_i \times 1$ response vector $\boldsymbol{y}$ for cluster i ($i = 1, 2, \ldots, N$):

$$\boldsymbol{y}_i = \boldsymbol{X}_i \boldsymbol{\beta} + \upsilon_i + \boldsymbol{\varepsilon}_i$$

$$\begin{aligned}
\boldsymbol{y}_i &= n_i \times 1 \text{ vector of responses for cluster } i \\
\boldsymbol{X}_i &= n_i \times (p+1) \text{ covariate matrix} \\
\boldsymbol{\beta} &= (p+1) \times 1 \text{ vector of regression coefficients} \\
\upsilon_i &= \text{cluster effects} \sim \mathcal{NID}(0, \sigma_\upsilon^2) \\
\boldsymbol{\varepsilon}_i &= n_i \times 1 \text{ vector of residuals} \sim \mathcal{NID}(0, \sigma^2 \boldsymbol{I}_{n_i})
\end{aligned}$$

- as cluster subscript i is present for $\boldsymbol{y}$ and $\boldsymbol{X}$, cluster sample size can vary
- the covariate matrix $\boldsymbol{X}$ can include
  - covariates measured at subject-level
  - covariates measured at cluster-level
  - cross-level interactions
- the total number of covariates $= p$
- the number of columns in $\boldsymbol{X}$ is $p + 1$ to include intercept (first column of $\boldsymbol{X}$ consists only of ones)

υ_i - random parameter distributed $\mathcal{NID}(0, \sigma_\upsilon^2)$

- distinguishes model from ususal (fixed-effects) multiple regression model

- represent effect of subject clustering (one for every cluster)

- if subject clustering has little effect
  - estimates of $\upsilon_i \approx 0$
  - σ_υ^2 will approach 0

- if subject clustering has strong effect
  - estimates of $\upsilon_i \neq 0$
  - σ_υ^2 will increase from 0

$$\boldsymbol{y}_i \sim \mathcal{NID}(\boldsymbol{X}_i\boldsymbol{\beta}, \sigma_v^2 \mathbf{1}_i \mathbf{1}_i' + \sigma^2 \boldsymbol{I}_{n_i})$$

- usual mean from multiple regression model
- var-covar structure accounts for clustering
  - within a cluster, variance $= \sigma^2 + \sigma_v^2$ and covariance $= \sigma_v^2$
  - "compound symmetry" structure
  - ratio of the cluster variance σ_v^2 to the total variance $\sigma^2 + \sigma_v^2$ is the *intraclass correlation*

Intra-"class" correlation $r = \sigma_v^2/(\sigma_v^2 + \sigma^2)$

- "class" is bad term, since in education "class" has meaning
- Goldstein suggests "intra-unit" correlation, replacing "unit" with appropriate term (clinic, school, family, firm *etc.*,)
- takes on values between 0 (when $\sigma_v^2 = 0$) and 1 (when $\sigma^2 = 0$)
- degree of similarity of measurements within a cluster
- ratio of variability attributable to cluster over total variability
- proportion of total (unexplained) variability of y_{ij} which is accounted for the clusters
- tends larger for smaller clusters (Kish, 1965; Donner, 1982)
  - 0.05 to 0.12 for spouse pairs, 0.0016 to 0.0126 for physician practices, 0.0005 to 0.0085 for counties
- can change depending on the dependent variable

Anorexic Women Study (Casper) - 63 sisters in 26 families
Maximum Likelihood (ML) estimates

	Height	Psych Factor	BMI
intercept	64.166	0.568	0.352
family variance	2.743	0.031	0.000
residual variance	2.895	0.055	0.005
intra-family correlation	0.487	0.362	0.000
descriptive statistics			
mean	64.16	0.57	0.35
variance	5.66	0.084	0.005

Random-effects Regression Models for Clustered Data: With an Example from Smoking Prevention Research

Hedeker, Gibbons, and Flay

Journal of Consulting and Clinical Psychology, 1994, 62:757-765

The Television School and Family Smoking Prevention and Cessation Project (Flay, *et al.*, 1988); a subsample of this project was chosen with the characteristics:

- *sample* - 1600 7th-graders - 135 classrooms - 28 LA schools
  - between 1 to 13 classrooms per school
  - between 2 to 28 students per classroom
- *outcome* - knowledge of the effects of tobacco use
- *timing* - students tested at pre and post-intervention
- *design* - schools randomized to
  - a social-resistance classroom curriculum (CC)
  - a media (television) intervention (TV)
  - CC combined with TV
  - a no-treatment control group

Tobacco and Health Knowledge Scale

Subgroup Descriptive Statistics at Pretest and Post-Intervention

	CC = no		CC = yes	
	TV = no	TV = yes	TV = no	TV = yes
n	421	416	380	383
Pretest mean	2.152	2.087	2.050	1.979
sd	1.182	1.288	1.285	1.286
Post-Int mean	2.361	2.539	2.968	2.823
sd	1.296	1.437	1.405	1.312
Difference	0.209	0.452	0.918	0.844

Within-Cluster / Between-Cluster representation

<u>*Within-clusters model*</u> *- level 1* $(j = 1, \ldots, n_i)$

$$PostTHKS_{ij} = b_{0i} \; + \; \left[b_{1i} PreTHKS_{ij}\right] \; + \; \varepsilon_{ij}$$

<u>*Between-clusters model*</u> *- level 2* $(i = 1, \ldots, N)$

$$b_{0i} \; = \; \beta_0 \; + \; [\beta_2 CC_i] \; + \; \upsilon_{0i}$$

$$b_{1i} \; = \; \beta_1$$

$$\varepsilon_{ij} \; \sim \; NID(0, \sigma^2) \quad \text{level-1 residuals}$$

$$\upsilon_{0i} \; \sim \; NID(0, \sigma_\upsilon^2) \quad \text{level-2 residuals}$$

TVSFP Study (Flay *et al.*, 1988): Tobacco and Health Knowledge *Posttest* Scores
1600 students in 135 classrooms in 28 schools: ML estimates (and standard errors)

	students in classrooms			*students in schools*		
Intercept	2.618	2.007	1.757	2.682	2.047	1.800
	(0.052)	(0.072)	(0.080)	(0.078)	(0.089)	(0.092)
Pretest score		0.302	0.310		0.303	0.310
		(0.026)	(0.026)		(0.026)	(0.026)
Classroom curriculum			0.497			0.470
			(0.086)			(0.106)
Cluster var	0.194	0.157	0.096	0.130	0.101	0.044
	(0.043)	(0.037)	(0.029)	(0.045)	(0.036)	(0.020)
Residual var	1.725	1.601	1.601	1.788	1.653	1.653
	(0.064)	(0.060)	(0.059)	(0.064)	(0.059)	(0.059)
ICC	0.101	0.090	0.057	0.068	0.057	0.026
$\log L$	-2760.9	-2696.4	-2681.3	-2756.8	-2692.0	-2684.7
χ^2_1		129.0	30.2		129.6	14.6

Within-Cluster / Between-Cluster representation

<u>Within-clusters model</u> - level 1 $(j = 1, \ldots, n_i)$

$$PostTHKS_{ij} = b_{0i} \; + \; \varepsilon_{ij}$$

<u>Between-clusters model</u> - level 2 $(i = 1, \ldots, N)$

$$b_{0i} = \beta_0 \; + \; \beta_1 CC_i \; + \; \beta_2 TV_i \; + \; \beta_3 (CC_i \times TV_i) \; + \; \upsilon_{0i}$$

$$\begin{aligned} \varepsilon_{ij} &\sim NID(0, \sigma^2) \quad \text{level-1 residuals} \\ \upsilon_{0i} &\sim NID(0, \sigma_\upsilon^2) \quad \text{level-2 residuals} \end{aligned}$$

- If cluster effect is completely explained by condition, then
  - $\upsilon_{0i} = 0$ for all i (thus $\sigma_\upsilon^2 = 0$)
  - model is same as ordinary regression (individual-level analysis)
- If $n_i = n$ for all clusters (and no level-1 covariates), then
  - model is same as ordinary regression of cluster means (cluster-level analysis)

THKS post-intervention scores - Regression estimates (se)

	Ordinary Regression		*Multilevel Model*
	Class-level	Student-level	Students in classes
Intercept	**2.342**	**2.361**	**2.341**
	(.117)	(.066)	(.092)
classroom curriculum (CC)	**.507**	**.607**	**.589**
	(.166)	(.096)	(.133)
television (TV)	-.082	.177	.120
	(.150)	(.094)	(.131)
interaction (CC by TV)	.011	*-.323*	-.247
	(.236)	(.137)	(.189)
residual variance	.468	1.860	**1.727**
			(.064)
class variance			**.134**
			(.037)
p < .05 **p < .01**			*ICC = .072*

Within-Cluster / Between-Cluster representation

<u>*Within-clusters model*</u> *- level 1* $(j = 1, \ldots, n_i)$

$$PostTHKS_{ij} = b_{0i} \; + \; b_{1i} PreTHKS_{ij} \; + \; \varepsilon_{ij}$$

<u>*Between-clusters model*</u> *- level 2* $(i = 1, \ldots, N)$

$$b_{0i} \; = \; \beta_0 \; + \; \beta_2 CC_i \; + \; \beta_3 TV_i \; + \; \beta_4 (CC_i \times TV_i) \; + \; \upsilon_{0i}$$

$$b_{1i} \; = \; \beta_1$$

$$\varepsilon_{ij} \; \sim \; NID(0, \sigma^2) \quad \text{level-1 residuals}$$

$$\upsilon_{0i} \; \sim \; NID(0, \sigma_\upsilon^2) \quad \text{level-2 residuals}$$

THKS Post-Intervention Scores - Regression Estimates (se)

	Ordinary Regression Models				*Multilevel Models*					
	Class-level		Student-level		Stu in classes		Stu in schools		Three-level	
Intercept	1.3087	***	1.6613	***	1.6776	***	1.6952	***	1.6970	***
	(0.229)		(0.084)		(0.099)		(0.115)		(0.117)	
pretest THKS	0.4962	***	0.3252	***	0.3116	***	0.3103	***	0.3072	***
	(0.097)		(0.026)		(0.026)		(0.026)		(0.026)	
classroom curriculum	0.5749	***	0.6406	***	0.6330	***	0.6601	***	0.6392	***
	(0.153)		(0.092)		(0.119)		(0.144)		(0.147)	
television	-0.0150		0.1987	**	0.1597		0.2023		0.1781	
	(0.150)		(0.090)		(0.117)		(0.140)		(0.144)	
interaction	-0.0485		-0.3216	**	-0.2747		-0.3696	*	-0.3204	
	(0.216)		(0.130)		(0.168)		(0.201)		(0.206)	
error variance	0.3924		1.6929		1.6030	***	1.6522	***	1.6020	***
					(0.059)		(0.059)		(0.059)	
class variance					0.0870	***			0.0636	**
					(0.028)				(0.028)	
school variance							0.0372	**	0.0258	
							(0.018)		(0.020)	

$^{***}p < 0.01$ $^{**}p < 0.05$ $^{*}p < 0.10$

Results

- conclusions about CC by TV interaction differ
  - non-significant by class-level analysis, significant by student-level analysis, marginally significant by multilevel
- student-level results close to multilevel, but estimates are more similar than standard errors $\rightarrow$ underestimation of standard errors by ordinary regression analysis is expected since assumption of independence of observations is violated
- students more homogeneous within classrooms than schools
  - students within classrooms model, $r = 0.052$
  - students within schools model, $r = 0.022$
- 3-level model close to students within classrooms model
  - based on 3-level model, classroom and school effects accounted for 3.8% and 1.5% of total variance, respectively

3-level ICCs

From the three-level model:
error var = 1.6020, class var = 0.0636, school var = 0.0258

Similarity of students within the same school

$$ICC = \frac{0.0258}{1.6020 + 0.0636 + 0.0258} = .0153$$

Similarity of students within the same classrooms (and schools)

$$ICC = \frac{0.0636 + 0.0258}{1.6020 + 0.0636 + 0.0258} = .0529$$

Similarity of classes within the same school

$$ICC = \frac{0.0258}{0.0636 + 0.0258} = .289$$

Explained Variance (Hox, *Multilevel Analysis*, 2002)

$$\text{level-1} \quad R_1^2 = 1 - \frac{\hat{\sigma}_p^2}{\hat{\sigma}_0^2} \qquad\qquad \text{level-2} \quad R_2^2 = 1 - \frac{\hat{\sigma}_{\upsilon_p}^2}{\hat{\sigma}_{\upsilon_0}^2}$$

subscript 0 refers to a model with no covariates (*i.e.*, null model), subscript p refers to a model with p covariates (*i.e.*, full model)

e.g., students in classrooms models

		models		
level	variance	null	full	R^2
1 (students)	$\hat{\sigma}^2$	1.725	1.603	.071
2 (classrooms)	$\hat{\sigma}_\upsilon^2$	.194	.087	.552

Explained Variance: 3-level model

$$R_1^2 = 1 - \frac{\hat{\sigma}_p^2}{\hat{\sigma}_0^2} \qquad R_2^2 = 1 - \frac{\hat{\sigma}_{v_{(2)p}}^2}{\hat{\sigma}_{v_{(2)0}}^2} \qquad R_3^2 = 1 - \frac{\hat{\sigma}_{v_{(3)p}}^2}{\hat{\sigma}_{v_{(3)0}}^2}$$

subscript 0 refers to a model with no covariates (*i.e.*, null model),
subscript p refers to a model with p covariates (*i.e.*, full model)

e.g., students in classrooms in schools models

level	variance	null	full	R^2
1 (students)	$\hat{\sigma}^2$	1.724	1.602	.071
2 (classrooms)	$\hat{\sigma}_{v_{(2)}}^2$	.085	.064	.247
3 (schools)	$\hat{\sigma}_{v_{(3)}}^2$	.110	.026	.764

Likelihood-ratio tests:
suppose Model I is nested within Model II

$$2 \times \log(L_{\mathrm{II}} \,/\, L_{\mathrm{I}}) \;=\; 2 \times (\log L_{\mathrm{II}} - \log L_{\mathrm{I}}) \;\sim\; \chi^2_q$$

where q = number of additional parameters in Model II

$-2 \log L$ is called the *deviance* (the higher the deviance the poorer the model fit)

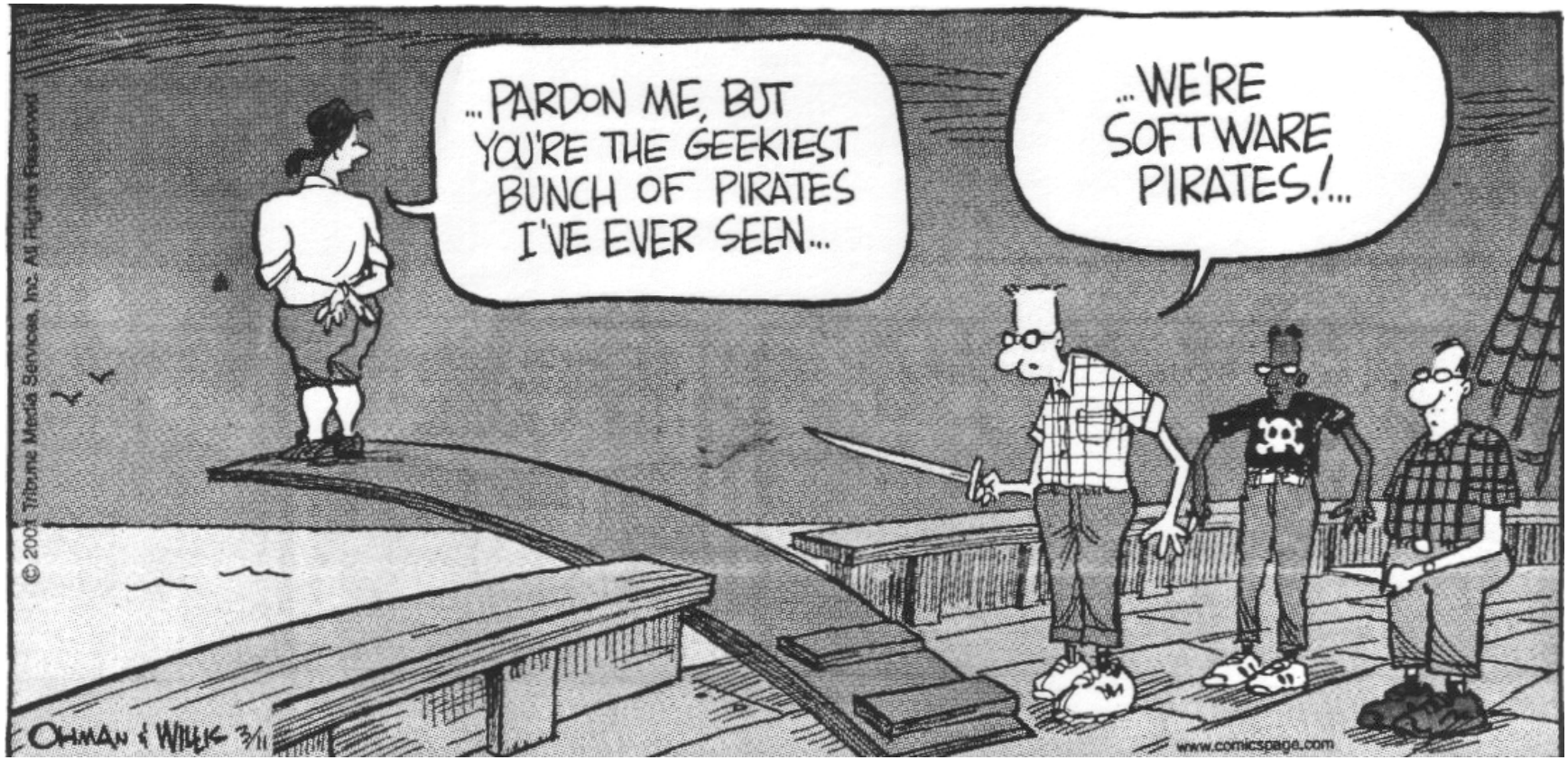
$$D_{\mathrm{I}} - D_{\mathrm{II}} \;\sim\; \chi^2_q$$

to evaluate the null hypothesis that the additional parameters in Model II jointly equal 0

Comparison of models using LR tests

Model	deviance	CM	χ^2	df	$p <$	halved $p <$
1. student-level	5377.90					
2a. students in classes	5359.96	1	17.94	1	.001	.001
2b. students in schools	5366.01	1	11.89	1	.001	.001
3. three-level	5357.36	1	20.54	2	.001	.001
		2a	2.60	1	.107	.053

LR tests with halved p-values (akin to one-tailed p-values) for tests of variance and covariance parameters is recommended (see Snijders & Bosker, *Multilevel Analysis*, 1999, pps. 90-91)


...PARDON ME, BUT YOU'RE THE GEEKIEST BUNCH OF PIRATES I'VE EVER SEEN...
...WE'RE SOFTWARE PIRATES!...
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OHMAN & WILLIS 3/11
www.comicspage.com

Software for Mixed Models

SAS

- Singer, J. D. (1998). Using SAS PROC MIXED To Fit Multilevel Models, Hierarchical Models, and Individual Growth Models. *Journal of Educational and Behavioral Statistics, 23*, 323-355.
- Singer, J. D. (2002). Fitting individual growth models using SAS PROC MIXED. In D. S. Moskowitz & S. L. Hershberger (Eds.), Modeling intraindividual variability with repeated measures data: Methods and applications (pp. 135-170). Mahwah, NJ: Lawrence Erlbaum Associates.

SPSS

- Peugh, J. L. and Enders, C. K. (2005). Using the SPSS Mixed Procedure to Fit Cross-Sectional and Longitudinal Multilevel Models. *Educational and Psychological Measurement, 65*, 717-741.
- Painter, J. Notes on using SPSS Mixed Models. http://www.unc.edu/∼painter/SPSSMixed/SPSSMixedModels.PDF

SAS code for regression and multilevel analysis: tvsfpmix.sas

```
OPTIONS NOCENTER;
TITLE1 'Analysis of TVSFP data:  Regression of Post THKS scores';

DATA tvsfp;
INFILE 'C:\MIX\tvsfp2b.dat';
INPUT schoolid classid posthks int prethks cc tv cctv;

PROC FORMAT;
VALUE cc 0='no' 1='yes' ;
VALUE tv 0='no' 1='yes' ;

/* student-level OLS analysis ignoring clustering */
PROC REG;
MODEL posthks = prethks cc tv cctv;
TITLE2 'OLS Student-level analysis ignoring clustering';

/* student-level ML analysis ignoring clustering */
PROC MIXED METHOD=ML COVTEST;
MODEL posthks = prethks cc tv cctv / SOLUTION;
TITLE2 'ML Student-level anaylsis ignoring clustering';
```

```
/* 2-level:  students nested within classrooms analysis */
PROC MIXED METHOD=ML COVTEST;
CLASS classid;
MODEL posthks = prethks cc tv cctv / SOLUTION;
RANDOM INTERCEPT / SUB=classid;
TITLE2 '2-level:  students nested within classrooms analysis';

/* 2-level:  students nested within schools analysis */
PROC MIXED METHOD=ML COVTEST;
CLASS schoolid;
MODEL posthks = prethks cc tv cctv / SOLUTION;
RANDOM INTERCEPT / SUB=schoolid;
TITLE2 '2-level:  students nested within schools analysis';

/* 3-level:  students in classrooms in schools analysis */
PROC MIXED METHOD=ML COVTEST;
CLASS classid schoolid;
MODEL posthks = prethks cc tv cctv / SOLUTION;
RANDOM INTERCEPT / SUB=schoolid;
RANDOM INTERCEPT / SUB=classid(schoolid);
TITLE2 '3-level:  students in classrooms in schools analysis';
RUN;
```

SPSS MIXED code - TVSFPC.SPS - after opening TVSFP.SAV
(SPSS dataset with variables: schoolid, classid, postthks, prethks, cc, tv, cctv)

```
* 2-level:  students nested within classrooms analysis .
MIXED
postthks WITH prethks cc tv cctv
/FIXED = prethks cc tv cctv
/METHOD = ML
/PRINT = SOLUTION TESTCOV
/RANDOM INTERCEPT | SUBJECT(classid) .


For 2-level:  students nested within schools analysis use:
/RANDOM INTERCEPT | SUBJECT(schoolid) .

For 3-level:  students in classrooms n schools analysis use:
/RANDOM INTERCEPT | SUBJECT(schoolid)
/RANDOM INTERCEPT | SUBJECT(schoolid*classid) .
```

- code and dataset available at http://www.uic.edu/∼hedeker/ml.html
- `METHOD=ML` or `/METHOD=ML` requests maximum likelihood estimation
  - ML estimation yields biased estimates for variance parameters (too small), but only matters if sample size is small
  - REML estimation (the default) corrects this bias, but can't be used for comparing models with different covariates by likelihood-ratio tests
- `COVTEST` or `TESTCOV` requests "Wald tests" for (co)variance parameters, however
  - dubious due to reliance on normal sampling distribution; use as guide
  - instead, use LR test with halved p-values for (co)variance parameters